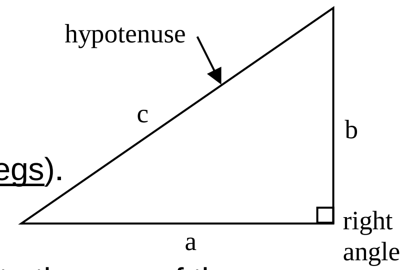
<u>Geometry: Pythagorean Theorem</u>—Explanation & Practice



In a **right triangle**, the side opposite the right angle (90°) is called the **hypotenuse**. The hypotenuse is the longest side.

The Greek mathematician Pythagoras discovered an important relationship between the hypotenuse and the two shorter sides (or <u>legs</u>). This relationship is called the **Pythagorean Theorem**.

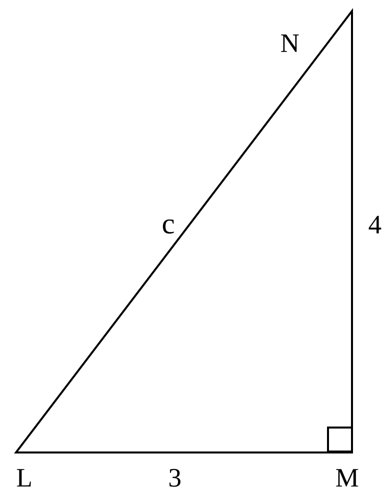
In words: <u>In a right triangle</u>, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

In symbols: $c^2 = a^2 + b^2$ (using the labels on the triangle above)

If you know the lengths of the two shorter sides (legs), you can use the Pythagorean Theorem to find the length of the hypotenuse.

Example

What is the length of the hypotenuse of ΔLMN?



Step 1. Find the square of each shorter side.
Substitute 3 for a and find a^2
$a^2 = 3^2 = 9$
Substitute 4 for b and find b^2
$b^2 = 4^2 = 16$

Step 2. Add the squares found in Step 1.
$c^2 = a^2 + b^2$
$= 9 + 16$
$= 25$

Step 3. To find c, take the square root of 25.
$c = \sqrt{25} = 5$

Answer: The length of the hypotenuse is 5 feet.

The Pythagorean Theorem can be written so that you can find the unknown length of one side of a right triangle if you know the lengths of the hypotenuse and the other side.

In symbols: $a^2 = c^2 - b^2$ (also $b^2 = c^2 - a^2$)

In words: Side a squared equals the hypotenuse squared minus side b squared.

Example:

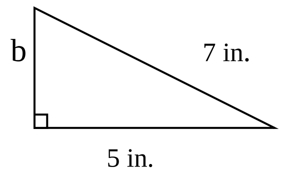
In the example above, if you know the hypotenuse is 5 and one leg is 4, you can find the other leg by subtracting squares:

$$a^2 = 5^2 - 4^2$$
$$= 25 - 16$$
$$= 9$$
$$\text{or } a = \sqrt{9} = 3$$

When using the Pythagorean Theorem, you may have to take the square root of a number which is not a perfect square. Use the calculator to get an approximation of such a square root.

Example:

Find the length of the leg b in the given right triangle. Give the exact length and a two-decimal place approximation.



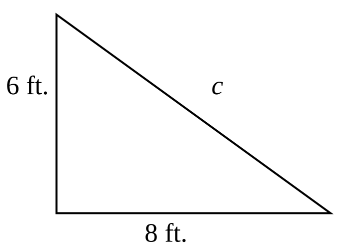
Let $a = 5$ and $c = 7$

$$b^2 = c^2 - a^2$$
$$b^2 = 7^2 - 5^2$$
$$b^2 = 49 - 25$$
$$b^2 = 24$$
$$\sqrt{b^2} = \sqrt{24}$$
$$b = \sqrt{24} \quad \text{(exact answer)}$$
$$b \approx 4.90 \quad \text{(approximate answer using a calculator)}$$

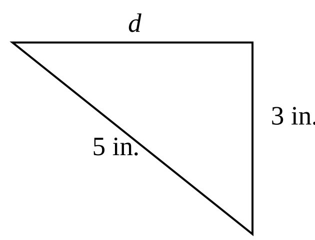
Practice

In 1-3, use the Pythagorean Theorem to find each unknown length.

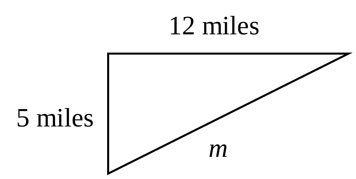
1.



2.

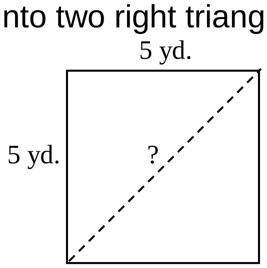


3.

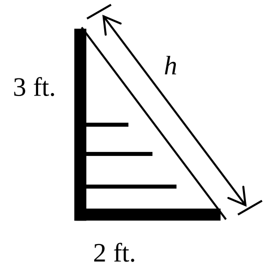


For approximate answers, round to the nearest tenth.

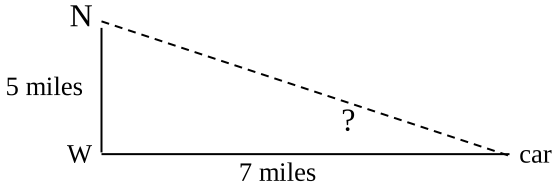
4. A **square** is a four-sided figure with four right angles and four equal sides. What is the approximate length of the diagonal that divides the square below into two right triangles?



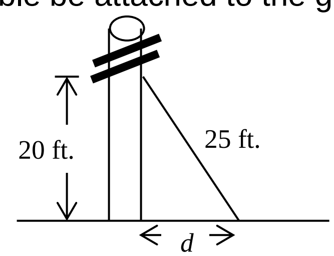
5. The end of a display case is in the shape of a right triangle. The base of the case is 2 feet wide, and the height of the case is 3 feet. What is the approximate height (h) of the glass front of the case?



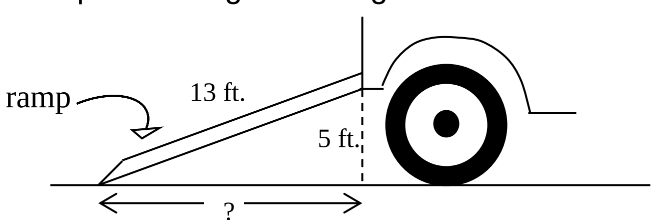
6. Lucinda hiked 7 miles west and 5 miles north during the first day of her camping trip. What is the approximate straight-line distance Lucinda is from her car?



7. A support cable is going to be attached to a telephone pole. The cable will be attached to the pole at a point of 20 feet above the ground. If the cable is 25 feet long, at what distance (*d*) from the pole will the cable be attached to the ground?

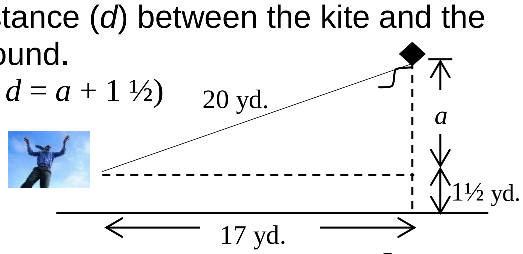


8. A mover's ramp is 13 feet long. If the bed of the truck is 5 feet above the ground, how far from the truck is the end of the ramp when the truck and ramp are sitting on level ground?



9. Lenny is flying a kite that's attached to the end of a string. Using the distances indicated in the drawing below, determine the approximate distance (*d*) between the kite and the ground.

(Hint: $d = a + 1\frac{1}{2}$)



Answer Key

1. c = 10 ft. ($\sqrt{36+64} = \sqrt{100}$)

2. d = 4 in. ($\sqrt{25-9} = \sqrt{16}$)

3. m = 13 mi. ($\sqrt{144+25} = \sqrt{169}$)

4. length $\approx$ 7.1 yd. ($\sqrt{25+25} = \sqrt{50}$)

5. $h \approx$ 3.6 ft. ($\sqrt{4+9} = \sqrt{13}$)

6. distance $\approx$ 8.6 miles ($\sqrt{25+49} = \sqrt{74}$)

7. d = 15 ft. ($\sqrt{625-400} = \sqrt{225}$)

8. distance = 12 ft. ($\sqrt{169-25} = \sqrt{144}$)

9. $d \approx$ 12 yd. $\sqrt{400-289} + 1\frac{1}{2} \approx 10.5 + 1.5$