

Order of Operations—Explanation & Practice

The **properties of real numbers** are often used to rewrite algebraic expressions.

The Order of Operations Agreement

To simplify an expression with more than one operation follow these steps:

- | | |
|---------------|---|
| Step 1 | Perform operations inside grouping symbols. Grouping symbols include parentheses (), brackets [], braces { }, the fraction bar, and the absolute value symbol $ $. |
| Step 2 | Simplify exponential expressions. |
| Step 3 | Perform multiplication and division as they occur from <u>left to right</u> . |
| Step 4 | Perform addition and subtraction as they occur from <u>left to right</u> . |

Use the saying **Please Exercise My Dear Aunt Sally to remember the order of operations:**

Please: **Parentheses** – Perform the operations within grouping symbols first (parentheses, fraction bar, etc.), in the order given in steps 2, 3, and 4.

Exercise: **Exponents** – Perform the operations indicated by exponents.

My Dear: **Multiply and Divide** – Perform only multiplication and division as they appear from left to right.

Aunt Sally: **Add and Subtract** – Perform addition and subtraction as they appear from left to right.

Simplify: $12 - 24(8 - 5) \div 2^2$

- | | |
|---------------------------------------|--|
| $12 - 24(8 - 5) \div 2^2$ | Step 1) Perform operations inside grouping symbols |
| \downarrow
$12 - 24(3) \div 2^2$ | Step 2) Simplify exponential expressions. |
| \downarrow
$12 - 24(3) \div 4$ | Step 3) Perform multiplication and division as they occur from left to right |
| \downarrow
$12 - 72 \div 4$ | |
| \downarrow
$12 - 18$ | |
| $12 + (-18)$ | Step 4) Perform addition and subtraction as they occur from left to right. |
| -6 | |

One or more of the previous steps may not be needed to simplify an expression. In that case, proceed to the next step in the Order of Operations Agreement.

Simplify: $\frac{4+8}{2+1} - (3-1) + 2$

$$\begin{array}{rcl}
 \frac{4+8}{2+1} - (3-1) + 2 & \text{Perform operation inside grouping symbols.} \\
 \downarrow \quad \downarrow \\
 \frac{12}{3} - 2 + 2 & \text{Perform multiplication and division as they occur from left to right.} \\
 \downarrow \\
 4 - 2 + 2 & \text{Perform addition and subtraction as they occur from left to right.} \\
 4 + (-2) + 2 \\
 \downarrow \\
 2 + 2 \\
 \downarrow \\
 4
 \end{array}$$

When an expression has grouping symbols inside grouping symbols, perform the operations inside the inner grouping symbols first.

Simplify: $6 \div [4 - (6 - 8)] + 2^2$

$$\begin{array}{rcl}
 6 \div [4 - (6 - 8)] + 2^2 & \text{Perform operations inside grouping symbols.} \\
 \downarrow \\
 6 \div [4 - (-2)] + 2^2 \\
 \downarrow \\
 6 \div [4 + 2] + 2^2 \\
 \downarrow \\
 6 \div 6 + 2^2 & \text{Simplify exponential expressions.} \\
 \downarrow \\
 6 \div 6 + 4 \\
 \downarrow \\
 1 + 4 & \text{Perform multiplication and division from left to right.} \\
 \downarrow \\
 5 & \text{Perform addition and subtraction from left to right.}
 \end{array}$$

More Examples of Simplifying Expressions Using the Order of Operations

1) $8 - 10 \div 2$
 $8 - 5$
 3

Divide
 Subtract

2) $(6 - 4)(6)$
 $2(6)$
 12

Subtract inside parentheses
 Multiply

$$\begin{array}{r}
 3) \quad 54 \div 6 \bullet 3 \\
 \quad 9 \bullet 3 \\
 \quad 27
 \end{array}$$

Neither multiplication nor division takes precedence over the other, so perform the operations from left to right.

$$\begin{array}{r}
 4) \quad 7 \bullet 9 + 6 \bullet 2 \\
 \quad 63 + 12 \\
 \quad 75
 \end{array}$$

Multiply
Add

$$\begin{array}{r}
 5) \quad 25 - 6 \div 3 + 8 \bullet 4 \\
 \quad 25 - 2 + 32 \\
 \quad \quad 23 + 32 \\
 \quad \quad 55
 \end{array}$$

Divide and multiply
Subtract
Add

$$\begin{array}{r}
 6) \quad 5 \bullet 9 + 9 - 6 (7+1) \\
 \quad 5 \bullet 9 + 9 - 6 \bullet 8 \\
 \quad 45 + 9 - 48 \\
 \quad \quad 54 - 48 \\
 \quad \quad 6
 \end{array}$$

Add in () first
Multiply
Add
Subtract

$$\begin{array}{r}
 7) \quad 3 \bullet 4^3 - 8 \bullet 3^2 + 11 \\
 \quad 3 \bullet 64 - 8 \bullet 9 + 11 \\
 \quad \quad 192 - 72 + 11 \\
 \quad \quad \quad 120 + 11 \\
 \quad \quad \quad 131
 \end{array}$$

Exponents

Multiply
Subtract
Add

$$\begin{array}{r}
 8) \quad (2^2 + 2 \bullet 3)^2 + 3^2 \\
 \quad (2^2 + 2 \bullet 3)^2 + 3^2 \\
 \quad (4 + 2 \bullet 3)^2 + 3^2 \\
 \quad \quad (4 + 6)^2 + 3^2 \\
 \quad \quad \quad 10^2 + 3^2 \\
 \quad \quad \quad 100 + 9 \\
 \quad \quad \quad 109
 \end{array}$$

Perform operations inside parentheses using proper order:
Inside the parentheses: exponents
Inside the parentheses: multiply
Inside the parentheses: add
No more grouping symbols; note the exponents
Add

Practice

1. $(2 + 8) - (7 - 3)$
2. $5(6 - 4) + 2(8 - 5)$
3. $(7 + 3) \cdot 6 + 5$
4. $7 + (3 \cdot 6) + 5$
5. $4 \cdot 3 + 6 \cdot 5$
6. $4 \cdot 14 - 9 \div 3 + 6 \cdot 2$
7. $24 \div 6 + 6 - 3(5 - 3)$
8. $5 \cdot 2^3 - 2 \cdot 4^2 + 25 - 7 \cdot 3$
9. $(3^3 - 12 \div 4)^2 + 5^2$
10. $2 \cdot [4 + 3(7 - 2)]$
11. $3 + [2(16 + 9)]$
12. $[5(x + 2)] - 3x$
13. $(3x + 5) + 4(2x + 7)]$
14. $16x - [5(2x + 7)]$
15. $[37(6x - 5x)] - 35x$
16. $[4(2x - 5) + 7] + [3(x + 3) + 5x]$
17. $[7(x + 5) - 19] - [4(x - 6) + 10]$
18. $3\{[6(x - 2) + 4] - [2(2x - 5) + 6]\}$
19. $[(3 \cdot 2x) + 5] + \{4x - [7(x + 2)]\}$
20. $\frac{7+5}{8-2 \cdot 2} \cdot 2^2 + 3$

Answer Key

1. 6
2. 16
3. 65
4. 30
5. 42
6. 65
7. 4
8. 12
9. 601
10. 38
11. 53
12. $2x + 10$
13. $11x + 33$
14. $6x - 35$
15. $2x$
16. $16x - 4$
17. $3x + 30$
18. $6x - 12$
19. $3x - 9$
20. 15