<u>Basic Exponent Laws with Integer Exponents</u>—Explanation & Practice

Definitions: In this handout we deal only with expressions that have integers (positive or negative, and zero) as exponents.

Example: We study such expressions as

$$x^3 \qquad a^{-2} \qquad y^0 \qquad x^2y^3w^{-1} \qquad (m+n)^5$$

A positive *exponent* shows how many times the *base* is to be multiplied by itself.

Example: In the expression 3^4, 3 is the base and 4 is the exponent.

$$\text{base} \longrightarrow 3^4 \longleftarrow \text{exponent}$$

Its meaning is: $3^4 = 3(3)(3)(3)$ the value of which is 81.

In general,

Positive Integer Exponent	a^n means $\underbrace{a \cdot a \cdot a \cdot a \cdot \ldots \cdot a}_{n \text{ factors}}$

Common Error	An exponent applies *only* to the symbol directly before it. Thus $2x^2 \neq 2^2x^2$ but $(2x)^2 = 2^2x^2 = 4x^2$

Multiplying Powers: Let us multiply the quantity x^2 by x^3. From the explanation above, we know that $x^2 = x \cdot x$

and that $x^3 = x \cdot x \cdot x$

Multiplying, we obtain

$$\begin{aligned} x^2 \cdot x^3 &= (x \cdot x)(x \cdot x \cdot x) \\ &= x \cdot x \cdot x \cdot x \cdot x \\ &= x^5 \end{aligned}$$

Notice that the exponent in the result is the sum of the two original exponents.

$$x^2 \cdot x^3 = x^{2+3} = x^5$$

This will always be so **if the bases are the same**. We summarize this rule as our *first law of exponents*.

Multiplication of Like Bases	$x^a \cdot x^b = x^{a+b}$	Law #1

Example: $x^4(x^3) = x^{4+3} = x^7$

Every quantity having no exponent is understood to have an exponent of 1, even though it is not usually written.

The "invisible1" appears again. We saw it before as the unwritten coefficient of every term, and now as the unwritten exponent. It is also in the denominator.

$$x = \frac{1x^1}{1}$$

Do not forget about those "invisible 1's". We use them all the time.

Examples:

(a) $x(x^3) = x^{1+3} = x^4$

(b) $a^2(a^4)(a) = a^{2+4+1} = a^7$

(c) $x^a(x)(x^b) = x^{a+1+b}$

Dividing Powers: Let us divide x^5 by x^3.

$$\frac{x^5}{x^3} = \frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x} = \frac{x \cdot x \cdot x}{x \cdot x \cdot x} \cdot x \cdot x$$
$$= x \cdot x = x^2$$

The same result could have been obtained by *subtracting exponents*.

$$\frac{x^5}{x^3} = x^{5-3} = x^2$$

This always works **if the bases are the same**, and we state it as another law of exponents:

Dividing Like Bases	$\frac{x^a}{x^b} = x^{a-b} \quad (x \neq 0)$	Law #2

Example: Divide a^4b^3 by ab^2.

Solution: By Law #2, $\frac{a^4b^3}{ab^2} = a^{4-1}b^{3-2} = a^3b$

Raising a Power to a Power: Let us take a quantity raised to a power, say x^2, and raise the entire expression to another power, say 3.

$$(x^2)^3$$

By the definition of exponents, $(x^2)^3 = (x^2)(x^2)(x^2)$

$$= (x \cdot x)(x \cdot x)(x \cdot x)$$
$$= x \cdot x \cdot x \cdot x \cdot x \cdot x = x^6$$

a result that could have been obtained by *multiplying the exponents.*

$$(x^2)^3 = x^{2(3)} = x^6$$

In general,

A Power Of a Power	$(x^a)^b = x^{ab}$	Law #3

Examples:

(a) $(w^5)^2 = w^{5(2)} = w^{10}$

(b) $(a^{-3})^2 = a^{(-3)(2)} = a^{-6}$

(c) $(10^4)^3 = 10^{4(3)} = 10^{12}$

Raising a Product to a Power: We now raise a product, such as xy, to some power, say 3. $(xy)^3$

By the definition of exponents, $(xy)^3 = (xy)(xy)(xy)$

$$= x \cdot y \cdot x \cdot y \cdot x \cdot y$$
$$= x \cdot x \cdot x \cdot y \cdot y \cdot y$$
$$= x^3 y^3$$

In general,

The Power of a Product	$(xy)^n = x^n y^n$	Law #4

Examples:

(a) $(xyz)^5 = x^5 y^5 z^5$

(b) $(2x)^3 = 2^3 x^3 = 8x^3$

(c) $(3.5 \times 10^3)^2 = (3.5)^2 \times (10^3)^2 = 12.25 \times 10^6$

(d) $(3x^2 y^n)^3 = 3^3 (x^2)^3 (y^n)^3 = 27x^6 y^{3n}$

A good way to test a "rule" that you are not sure of is to try it with numbers. In this case, does $(2+3)^2$ *equal* 2^2+3^2 ? *Evaluating each expression, we obtain*

$$(5)^2 \stackrel{?}{=} 4 + 9$$
$$25 \neq 13$$

Common Error	There is *no* similar rule for the *sum* of two quantities raised to a power. $(x+y)^n \neq x^n + y^n$

Raising a Quotient to a Power: Using the same steps as in the preceding section, see if you can show that

$$\left(\frac{x}{y}\right)^3 = \frac{x^3}{y^3}$$

In general,

The Power of a Quotient	$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n} \quad (y \neq 0)$	Law #5

Examples:

(a) $\left(\frac{x}{5}\right)^2 = \frac{x^2}{5^2} = \frac{x^2}{25}$

(b) $\left(\frac{3a}{2b}\right)^3 = \frac{3^3a^3}{2^3b^3} = \frac{27a^3}{8b^3}$

(c) $\left(\frac{2x^2}{5y^3}\right)^3 = \frac{2^3(x^2)^3}{5^3(y^3)^3} = \frac{8x^6}{125y^9}$

Zero as an Exponent: If we divide x^n by itself, we get, by Law #2,

$$\frac{x^n}{x^n} = x^{n-n} = x^0$$

But any expression divided by itself equals 1, so

Any quantity (except 0) raised to the zero power equals 1.

Zero Exponent	$x^0 = 1 \quad (x \neq 0)$	Law #6

Examples:

(a) $(xyz)^0 = 1$

(b) $3862^0 = 1$

(c) $(x^2 - 2x + 3)^0 = 1$

(d) $5x^0 = 5(1) = 5$

Negative Exponents: We now divide x^0 by x^a. By Law #2,

$$\frac{x^0}{x^a} = x^{0-a} = x^{-a}$$

Since $x^0 = 1$, we get

Negative Exponents	$x^{-a} = \frac{1}{x^a} \quad (x \neq 0)$	Law #7

Examples:

(a) $5^{-1} = \frac{1}{5}$

(b) $x^{-4} = \frac{1}{x^4}$

(c) $\frac{1}{x^{-a}} = x^a$

(d) $\frac{1}{xy^{-2}} = \frac{y^2}{x}$

(e) $\frac{w^{-3}}{z^{-2}} = \frac{z^2}{w^3}$

Practice

Evaluate each expression.

1. $5^3 =$
2. $(-2)^2 =$
3. $(-6)^3 =$
4. $(9)^2 =$
5. $(-5)^4 =$
6. $\left(\frac{2}{3}\right)^2 =$
7. $(-10)^3 =$
8. $(-3)^3 =$
9. $5^4 =$
10. $\left(-\frac{4}{5}\right)^3 =$
11. $(5)^2 =$
12. $(-2)^3 =$
13. $8^2 - 3^2 =$
14. $9 - (3)^2 + (4)^4 =$
15. $7^2 + 3^3 - 4 =$
16. $5^0 + (-3)^2 + 4 =$
17. $4^3 - (3)^2 + (5)^0 =$
18. $\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 =$
19. $(-4)^3 - (-3)^3 + 2^4 =$
20. $\left(\frac{3}{4}\right)^2 - \left(-\frac{1}{2}\right)^2 + \frac{1}{2} =$
21. $5^2 \cdot (-2)^2 \div 2^2 =$
22. $(-3)^3 \div 3 \cdot 2^4 =$
23. $(-7)^0 \div 8 \cdot (-2)^3 =$
24. $\frac{(-3)^4 - 2^5}{7^2} \cdot 2^3$
25. 2^4
26. $(-3)^2$
27. $(-6)^3$
28. $(|-3|)^3$
29. $(0.1)^4$
30. $(-2)^5$

Multiply.

31. $a^3 \cdot a^5$
32. $x^a \cdot x^2 \cdot x^4$
33. $y^{a+1} \cdot y^{a-3}$
34. $10^4 \cdot 10^3$
35. $10^a \cdot 10^b$
36. $10^{n+2} \cdot 10^{2n-1}$

Divide. Write your answers without negative exponents.

37. $\frac{y^5}{y^2}$
38. $\frac{5^5}{5^3}$
39. $\frac{x^{n+2}}{x^{n+1}}$
40. $\frac{10^5}{10}$
41. $\frac{10^{x+5}}{10^{x+3}}$
42. $\frac{10^2}{10^{-3}}$

43. $\dfrac{x^{-2}}{x^{-3}}$

44. $\dfrac{a^{-5}}{a}$

Simplify.

45. $(x^3)^4$

46. $(9^2)^3$

47. $(a^x)^y$

48. $(x^{-2})^{-2}$

49. $(x^{a+1})^2$

Raise to the power indicated.

50. $(xy)^2$

51. $(2x)^3$

52. $(3x^2y^3)^2$

53. $(3abc)^3$

54. $\left(\dfrac{3}{5}\right)^2$

55. $\left(-\dfrac{1}{3}\right)^3$

56. $\left(\dfrac{x}{y}\right)^5$

57. $\left(\dfrac{2a}{3b^2}\right)^3$

58. $\left(\dfrac{3x^2y}{4wz^3}\right)^2$

Write each expression with positive exponents only.

59. a^{-2}

60. $(-x)^{-3}$

61. $\left(\dfrac{3}{y}\right)^{-3}$

62. $a^{-2}bc^{-3}$

63. $\left(\dfrac{2a}{3b^3}\right)^{-2}$

64. xy^{-4}

65. $2x^{-2} + 2y^{-3}$

66. $\left(\dfrac{x}{y}\right)^{-1}$

Express without fractions, using negative exponents where needed.

67. $\dfrac{1}{x}$

68. $\dfrac{3}{y^2}$

69. $\dfrac{x^2}{y^2}$

70. $\dfrac{x^2y^{-3}}{z^{-2}}$

71. $\dfrac{a^{-3}}{b^2}$

72. $\dfrac{x^{-2}y^{-3}}{w^{-1}z^{-4}}$

Simplify.

73. $(a+b+c)^0$

74. $8x^0y^2$

75. $\dfrac{a^0}{9}$

76. $\dfrac{y}{x^0}$

77. $\dfrac{x^{2n} \cdot x^3}{x^{3+2n}}$

78. $5\left(\dfrac{x}{y}\right)^0$

Answer Key
Exponent Laws: with Integer Exponents – Explanation & Practice

1. 125
2. 4
3. −216
4. 81
5. 625
6. $\frac{4}{9}$
7. −1,000
8. −27
9. 625
10. $-\frac{64}{125}$
11. 25
12. −8
13. 55
14. 256
15. 72
16. 14
17. 56
18. $\frac{3}{8}$
19. −21
20. $\frac{13}{16}$
21. 25
22. −144
23. −1
24. 8
25. 16
26. 9
27. -216
28. $(|-3|)^3=(3)^3 = 27$
29. 0.0001
30. -32
31. a^8
32. x^{a+6}
33. y^{2a-2}
34. 10^7
35. 10^{a+b}
36. 10^{3n+1}
37. y^3
38. $5^2=25$
39. X
40. 10^4
41. 10^2
42. 10^5
43. x
44. $\frac{1}{a^6}$
45. x^{12}
46. 9^6
47. a^{xy}
48. x^4
49. x^{2a+2}
50. $x^2\ y^2$
51. $8x^3$
52. $9x^4y^6$
53. $27a^3b^3c^3$
54. $\frac{9}{25}$
55. $-\frac{1}{27}$
56. $\frac{x^5}{y^5}$
57. $\frac{8a^3}{27b^6}$
58. $\frac{9x^4y^2}{16w^2z^6}$
59. $\frac{1}{a^2}$
60. $-\frac{1}{x^3}$
61. $\frac{y^3}{27}$
62. $\frac{b}{a^2c^3}$
63. $\frac{9b^6}{4a^2}$
64. $\frac{x}{y^4}$
65. $\frac{2}{x^2}+\frac{3}{y^3}$
66. $\frac{y}{x}$
67. x^{-1}
68. $3y^{-2}$
69. x^2y^{-2}
70. $x^2y^{-3}z^2$
71. $a^{-3}b^{-2}$
72. $wx^{-2}y^{-3}z^4$
73. 1
74. $8y^2$
75. $\frac{1}{9}$
76. y
77. 1
78. 5