

Radicals: Definitions & Simplification— Explanation & Practice

Definition: The **square** of a number is the product of that number times itself.
Product means multiply.

Example: $4 \cdot 4 = 16$

The square of 4 is 16, or 16 is the square of 4 (also written as $4^2 = 16$ or $16 = 4^2$).

The n^{th} power of a number is the product of that number times itself n times. For example: $2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$

$n^0 = 1$ as long as $n \neq 0$. Examples: $x^0 = 1$, $23^0 = 1$, $(y^2 + 3)^0 = 1$.

Number	X^1	X^2	X^3	X^4	X^5	X^6	X^7	X^8
1	1	1	1	1	1	1	1	1
2	2	4	8	16	32	64	128	256
3	3	9	27	81	243	729	2187	
4	4	16	64	256	1024			
5	5	25	125	625	3125			
6	6	36	216	1296				
7	7	49	343	2401				
8	8	64	512					
9	9	81	729					
10	10	100	1000					
11	11	121	1331					
12	12	144	1728					
13	13	169	2197					
14	14	196	2744					
15	15	225	3375					
16	16	256	4096					

Problem Set 1

Solve:

1. $2^2 = \underline{\hspace{2cm}}$

6. $6^0 = \underline{\hspace{2cm}}$

2. $4^3 = \underline{\hspace{2cm}}$

7. $3^3 = \underline{\hspace{2cm}}$

3. $6^2 = \underline{\hspace{2cm}}$

8. $2^7 = \underline{\hspace{2cm}}$

4. $5^4 = \underline{\hspace{2cm}}$

9. $7^2 = \underline{\hspace{2cm}}$

5. $13^2 = \underline{\hspace{2cm}}$

10. $15^3 = \underline{\hspace{2cm}}$

Problem Set 1—Answers:

1. 4
2. 64
3. 36
4. 625
5. 169
6. 1
7. 27
8. 128
9. 49
10. 3375

Problem Set 1, 1-5 explained:

1. The square of 2 equals $2 \cdot 2 = 4$
 2. The cube of 4 equals $4 \cdot 4 \cdot 4 = 64$
 3. 6^2 means find the square of 6 which equals 36
 4. $5^4 = 5 \cdot 5 \cdot 5 \cdot 5 = 625$
 5. $13^2 = 13 \cdot 13 = 169$
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Definition: The **square root** of a number is the number that when squared equals the first.
This symbol $\sqrt{\quad}$ means to find the square root.

The symbol and number together are called a radical expression.

Definition: The **n^{th} root** of x is the number that when raised to the n^{th} power equals the number under the radical. $\sqrt[n]{x} = y$ is equivalent to $y^n = x$. y is the n^{th} root of x

Example 1: $\sqrt{36} = 6$ The square root of 36 is 6, because 6 squared equals 36.

Example 2: $\sqrt[3]{343} = 7$ The cube root of 343 is 7 because the cube of 7 = 343 ($7 \cdot 7 \cdot 7 = 343$)

Problem Set 2: Find the square root of 9 _____
Find the square root of 16 _____
Find $\sqrt[3]{8}$ _____
Find $\sqrt{49}$ _____
2 is the _____ of 4

Extra Problems:

$\sqrt{1} =$	$4^2 =$
$\sqrt[3]{216} =$	$11^2 =$
$\sqrt{225} =$	$1^{12} =$
$\sqrt[3]{125} =$	$12^2 =$

Answer Key Set 2:

The square of 3 is $(3 \cdot 3)$ or 9; therefore, the square root of 9 is 3.

The square of 4 is 16; therefore, the square root of 16 is 4.

$\sqrt[3]{8}$ means find the cube root of 8. Since $2 \cdot 2 \cdot 2 = 8$, $\sqrt[3]{8} = 2$

$\sqrt{49}$ means find the square root of 49. Since $7 \cdot 7 = 49$, $\sqrt{49} = 7$.

2 is the square root of 4

Answers to Extra Problems:

1	16
6	121
15	1
5	144

To simplify radicals, we look for a power of the type radical as a factor inside the radical.

Example 1. $\sqrt{90} = \sqrt{9 \cdot 10} = 3\sqrt{10}$ since $\sqrt{9} = 3$ **Example 2.** $\sqrt{162} = \sqrt{81 \cdot 2} = 9\sqrt{2}$.

Simplifying Numerical Radicals—Practice Problems

- | | | | |
|-------------------|------------------|-------------------|-----------|
| 1. $\sqrt{25}$ | 2. $\sqrt{121}$ | 3. $\sqrt{169}$ | 1. _____ |
| | | | 2. _____ |
| | | | 3. _____ |
| | | | 4. _____ |
| | | | 5. _____ |
| 4. $\sqrt{196}$ | 5. $\sqrt{289}$ | 6. $\sqrt{400}$ | 6. _____ |
| | | | 7. _____ |
| 7. $\sqrt{18}$ | 8. $\sqrt{48}$ | 9. $\sqrt{20}$ | 8. _____ |
| | | | 9. _____ |
| 10. $\sqrt{45}$ | 11. $\sqrt{72}$ | 12. $3\sqrt{12}$ | 10. _____ |
| | | | 11. _____ |
| | | | 12. _____ |
| 13. $-6\sqrt{25}$ | 14. $9\sqrt{40}$ | 15. $\sqrt{125}$ | 13. _____ |
| | | | 14. _____ |
| | | | 15. _____ |
| 16. $-7\sqrt{50}$ | 17. $\sqrt{192}$ | 18. $3\sqrt{144}$ | 16. _____ |
| | | | 17. _____ |
| | | | 18. _____ |
| 19. $\sqrt{150}$ | 20. $\sqrt{900}$ | 21. $\sqrt{200}$ | 19. _____ |
| | | | 20. _____ |
| | | | 21. _____ |

- | | | | |
|-------------------|------------------|------------------|-----------|
| 22. $3\sqrt{160}$ | 23. $5\sqrt{98}$ | 24. $\sqrt{88}$ | 22. _____ |
| | | | 23. _____ |
| | | | 24. _____ |
| 25. $\sqrt{135}$ | 26. $\sqrt{128}$ | 27. $3\sqrt{75}$ | 25. _____ |
| | | | 26. _____ |
| | | | 27. _____ |
| 28. $\sqrt{700}$ | 29. $\sqrt{245}$ | 30. $2\sqrt{96}$ | 28. _____ |
| | | | 29. _____ |
| | | | 30. _____ |

Simplifying Numerical Radicals—Answer Key

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|-----------------|-------------------|-------------------|
| 1. 5 | 11. $6\sqrt{2}$ | 21. $10\sqrt{2}$ |
| 2. 11 | 12. $6\sqrt{3}$ | 22. $12\sqrt{10}$ |
| 3. 13 | 13. -30 | 23. $35\sqrt{2}$ |
| 4. 14 | 14. $18\sqrt{10}$ | 24. $2\sqrt{22}$ |
| 5. 17 | 15. $5\sqrt{5}$ | 25. $3\sqrt{15}$ |
| 6. 20 | 16. $-35\sqrt{2}$ | 26. $8\sqrt{2}$ |
| 7. $3\sqrt{2}$ | 17. $8\sqrt{3}$ | 27. $15\sqrt{3}$ |
| 8. $4\sqrt{3}$ | 18. 36 | 28. $10\sqrt{7}$ |
| 9. $2\sqrt{5}$ | 19. $5\sqrt{6}$ | 29. $7\sqrt{5}$ |
| 10. $3\sqrt{5}$ | 20. 30 | 30. $8\sqrt{6}$ |