Test 3 Study Guide Solutions

1. The normal curve with a mean of 10 would be to the left of a normal curve with a mean of 20.
Changing the mean shifts the curve left or right.

2. The normal curve with a standard deviation of 15 would be a wider, shorter curve than the normal curve with standard deviation 8.
The larger the standard deviation, the more spread out the data, creating a wider, shorter bell.

3. $\mu \pm 1\sigma$: $\overset{\mu=0,\ \sigma=1}{P(-1 \le z \le 1)} = P(z \le 1) - P(z \le -1)$
$= .8413 - .1587$
$= .6826$

$\boxed{68\%}$ within 1 standard deviation of the mean

$\mu \pm 2\sigma$: $\overset{\mu=0,\ \sigma=1}{P(-2 \le z \le 2)} = P(z \le 2) - P(z \le -2)$
$= .9772 - .0228$
$= .9544$

$\boxed{95\%}$ within 2 standard deviations of the mean

$\mu \pm 3\sigma$: $P(-3 \le z \le 3) = P(z \le 3) - P(z \le -3)$
$= .9987 - .0013$
$= .9974$

$\boxed{99.7\%}$ within 3 standard deviations of the mean

4. a. Area under curve from $z=1.36$ to $z=2.45$

$.9929 - .9131 = \boxed{.0798}$

b. Area under curve to left of $z=-1.11$

$\boxed{.1335}$

c. Area under curve to right of $z=-.09$

$1 - .4641 = \boxed{.5359}$

5. a. $P(z=2.39) = \boxed{0}$

b. $P(-2.15 < z < 2.15) = P(z<2.15) - P(z<-2.15)$

$= .9842 - .0158$

$= \boxed{.9684}$

c. $P(z>2.69) = 1 - P(z<2.69) = 1 - .9964$

$= \boxed{.0036}$

6. $\mu = 3 \quad \sigma = .009$

$z = \frac{2.98-3}{.009} = -2.22$

$P(x \leq 2.98) + P(x \geq 3.02)$

$z = \frac{3.02-3}{.009} = 2.22$

$= P(z \leq -2.22) + P(z \geq 2.22)$

$= P(z \leq -2.22) + (1 - P(z \leq 2.22))$

$= .0132 + (1 - .9868)$

$= .0264$

2.64% of nails produced are unusable.

* Alternative method:

$1 - P(2.98 \leq x \leq 3.02)$

$= 1 - (P(x \leq 3.02) - P(x \leq 2.98))$

$= 1 - (.9868 - .0132)$

$= .0264$

7. $\mu = 190 \quad \sigma = 21$ $\qquad z = \frac{x-\mu}{\sigma} = \frac{160-190}{21} = -1.43$

a. $P(x < 160)$

$= P(z < -1.43) = \boxed{.0764}$

b. $P(215 < x < 245)$ $\qquad z = \frac{215-190}{21} = 1.19$

$= P(1.19 < z < 2.62)$

$= P(z < 2.62) - P(z < 1.19)$ $\qquad z = \frac{245-190}{21} = 2.62$

$= .9956 - .8830$

$= \boxed{.1126}$

8. $\mu = 87 \quad \sigma = 22$ $\qquad z = \frac{70-87}{22} = -.77$

a. $P(70 < x\, 105)$

$= P(-.77 < z < .82)$ $\qquad z = \frac{105-87}{22} = .82$

$= P(z < .82) - P(z < -.77)$

$= .7939 - .2206$

$= \boxed{.5733}$

b. $P(x > 185)$ $\qquad z = \frac{185-87}{22} = 4.45$

$= P(z > 4.45)$

$= 1 - P(z < 4.45) = 1 - 1 = 0$ approximately

It is not likely a customer would spend more than \$185; however it is possible because the probability is approximately 0 (but not = 0) because the tails are assumed to be very close to 0.

9\. a. (Area under curve from 0 to z) = .1965

z is positive, so area under curve to left of z is .5 + .1965 = .6965 ← (falls between)

closer to → .6950   .6985

$\boxed{z = .51}$

b\. (Area under curve between 0 and z) = .2740

z is negative, so area under curve to left of z is .5 − .2740 = .2260

− closest to .2266

→ $\boxed{z = -.75}$

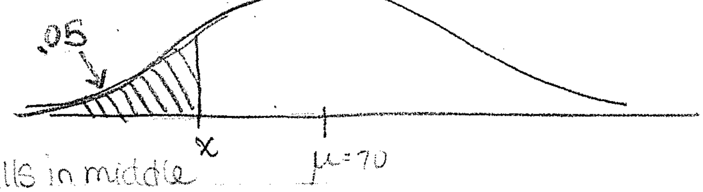
c\. (Area under the curve in the left tail) = .2050

$\boxed{z = -.82}$

d\. (Area under curve in right tail) = .1053

Area under curve left of z = 1 − .1053 = .8947

$\boxed{z = 1.25}$



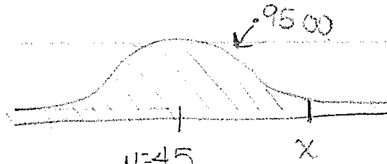
10\. $\mu = 70$ $\sigma = 8$

$x = 70 + (-1.64) \cdot 8$

$= 56.88$

falls in middle so $z = -1.64$ or -1.65

The warranty period should be about 57 months.



$z = 1.64$ or 1.65

11\. $\mu = 45$ $\sigma = 3$

$x = 45 + 1.64 \cdot 3$

$= 49.92$

It takes approximately 50 minutes to get to work 95% of the time so Ashley should leave by $\boxed{8{:}10 \text{ am}}$

12. a.

x	$P(x)$
2	.2
7	.4
8	.2
12	.2

b. ${}_5C_4 = 5$ possible samples of size 4

d.

Samples	$\bar{x}$	Sampling Error
7, 2, 7, 12	7	-.2
7, 2, 12, 8	7.25	.05
7, 7, 12, 8	8.5	1.3
7, 2, 7, 8	6	-1.2
2, 7, 12, 8	7.25	.05

$\bar{x}$	$P(\bar{x})$
6	.2
7	.2
7.25	.4
8.5	.2

c. $\mu = 7.2$

13. $\mu = 20.20$, $\sigma = 2.60$, $n = 18$

a. $\mu_{\bar{x}} = 20.20$

$\sigma_{\bar{x}} = \dfrac{2.60}{\sqrt{18}} = \boxed{.6128}$

The sampling distribution of $\bar{x}$ will be a normally shaped distribution.

b. i. $1 - P(19.2 < \bar{x} < 21.2)$

$= 1 - (P(z < 1.63) - P(z < -1.63))$

$= 1 - (.9484 - .0516)$

$= \boxed{.1032}$

$z = \dfrac{19.2 - 20.2}{.6128} = -1.63$

$z = \dfrac{21.2 - 20.2}{.6128} = 1.63$

ii. $P(20 \le \bar{x} \le 20.5)$

$= P(z \le .49) - P(z \le -.33)$

$= .6879 - .3707$

$= \boxed{.3172}$

$z = \dfrac{20 - 20.2}{.6128} = -.33$

$z = \dfrac{20.5 - 20.2}{.6128} = .49$

iii. $P(\bar{x} \ge 22)$

$= 1 - P(z \le 2.94)$

$= 1 - .9984 = \boxed{.0016}$

$z = \dfrac{22 - 20.2}{.6128} = 2.94$

$z = \dfrac{21 - 20.2}{.6128} = 1.31$

iv. $P(\bar{x} \le 21) = P(z \le 1.31)$

$= \boxed{.9049}$

14. $\mu = 28.2$, $\sigma = 6$, $n = 35$

$\boxed{\mu_{\bar{x}} = 28.2}$ $\sigma_{\bar{x}} = \frac{6}{\sqrt{35}} = \boxed{1.01}$

The shape of the sampling distribution of $\bar{x}$ would be a normally distributed curve.

15. a. $p = \frac{4}{6} = \boxed{.67}$

b. ${}_6C_5 = 6$ total samples

c.

Samples	$\hat{p}$	Sampling error (d.)
GGDDG	$3/5 = .6$	$-.07$
GGDDG	$3/5 = .6$	$-.07$
GGDGG	$4/5 = .8$	$.13$
GGDGG	$4/5 = .8$	$.13$
GDDGG	$3/5 = .6$	$-.07$
GDDGG	$3/5 = .6$	$-.07$

$\hat{p}$	$P(\hat{p})$
.6	.67
.8	.33

16. $p = .83$ $q = .17$ $n = 1000$

$np = 830 > 5$
$nq = 170 > 5$ so normal shape

a. $\boxed{\mu_{\hat{p}} = .83}$ $\sigma_{\hat{p}} = \sqrt{\frac{.83 \cdot .17}{1000}} = \boxed{.0119}$

$z = \frac{.82 - .83}{.0119} = -.84$

b. i $P(.82 \leq \hat{p} \leq .84) = P(-.84 \leq z \leq .84)$

$= .7995 - .2005 = \boxed{.599}$

$z = \frac{.85 - .83}{.0119} = 1.68$

ii. $P(\hat{p} > .85) = 1 - P(z \leq 1.68)$

$= 1 - .9535 = \boxed{.0465}$

$z = \frac{.81 - .83}{.0119} = -1.68$

iii $P(\hat{p} < .81) = P(z < -1.68) = \boxed{.0465}$

$\sigma_{\hat{p}} = \sqrt{\frac{.64 \cdot .36}{50}} = .0679$ $z = \frac{.54 - .64}{.0678} = -1.47$

17. $p = .64$ $P(.54 < \hat{p} < .61)$

$q = .36$ $= .3300 - .0708$

$n = 50$ $= \boxed{.2592}$

$z = \frac{.61 - .64}{.0678} = -.44$