# Solving Linear Equations: Review – Explanation

Why is it so important to solve equations? Because equations are used to describe the way certain things happen in the world. For example, the note produced by a guitar string is not a whim of nature, but can be predicted (solved for) when you know the length, the mass, and the tension in the string (when you know the *equation* relating the pitch, length, mass, and tension).

Thousands of equations exist that link together the various quantities in the physical world, in chemistry, finance, and so on, and their number is still increasing. To be able to solve and to manipulate such equations is essential for anyone who has to deal with these quantities on the job.

**EQUATIONS**

**Equations**: An equation has two *sides* or members, and an *equal sign*.

$$\underbrace{3x^2 - 4x}_{\text{left side}} = \underbrace{2x + 5}_{\text{right side}}$$

(↑ equal sign)

**Checking**: Check an apparent solution by substituting it back into the original equation.

**EXAMPLE:** Is the value $x = 17$ a solution to the equation $\frac{2x+1}{5} = 7$ ?

***Solution***: Substituting 17 for $x$ in the equation, we get

$$\frac{2(17)+1}{5} \stackrel{?}{=} 7$$

$$\frac{34+1}{5} \stackrel{?}{=} 7$$

$$\frac{35}{5} = 7$$ Checks!

> *Get into the habit of checking your work, for errors creep in everywhere.* But *even when you work correctly, you may sometimes get an answer that will not check. This is called an* extraneous solution.

| Common Error | Check your solution only in the *original* equation. Later versions may already contain errors. |
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**First-Degree Equations:** A *first-degree* equation (also called a *linear* equation) is one in which the terms containing the unknown are all of first degree.

**EXAMPLES:** The equations $2x + 3 = 9 - 4x$, $3x/2 = 6x + 3$, and $3x + 5y - 6z = 0$ are all of first degree.

**Solving an Equation:** To solve an equation, we *perform the same mathematical operation to both sides of the equation*. The object is to get the unknown standing alone on one side of the equation.

**EXAMPLE:** Solve $3x = 8 + 2x$.

***Solution:*** Subtracting $2x$ from both sides, we obtain

$$3x - 2x = 8 + 2x - 2x$$

Combining like terms yields

$$x = 8$$

When we subtracted $2x$ from both sides, it vanished from the right side of the equation and appeared on the left side as $(-2x)$. In general, any term can be moved to the other side of the equal sign, provided that you change its sign. This is called *transposing*.

**EXAMPLE:** Solve the equation $3x - 5 = x + 1$.

***Solution:*** We first subtract $x$ from both sides, and add 5 to both sides.

$$\begin{array}{rcr} 3x - 5 & = & x + 1 \\ \underline{-x + 5} & & \underline{-x + 5} \\ 2x & = & 6 \end{array}$$

Dividing both sides by 2, we obtain

$$x = 3$$

**Check** Substituting into the original equation yields

$$3(3) - 5 \stackrel{?}{=} 3 + 1$$

$$9 - 5 = 4 \qquad \text{Checks!}$$

**Symbols of Grouping:** When the equation contains symbols of grouping, remove them early in the solution.

**EXAMPLE:** Solve the equation $3(3x + 1) - 6 = 5(x - 2) + 15$.

***Solution:*** Removing the parentheses, we obtain

$$9x + 3 - 6 = 5x - 10 + 15$$

Combining like terms $\qquad 9x - 3 = 5x + 5$

Adding $-5x + 3$ to both sides, $\qquad \underline{-5x + 3} \qquad \underline{-5x + 3}$

$$4x = 8$$

Dividing by 4, $\qquad x = 2$

Check:

$$3(6+1) - 6 \stackrel{?}{=} 5(2-2) + 15$$

$$21 - 6 \stackrel{?}{=} 0 + 15$$

$$15 = 15 \qquad \text{Checks!}$$

| | |
|---|---|
| Common Error | The mathematical operations you perform must be done *to both sides* of the equation in order to preserve the equality. |
| Common Error | The mathematical operations you perform on both sides of an equation must be done to each side *as a whole*—not term by term. |

**Fractional Equations:** A *fractional equation* is one that contains one or more fractions. When an equation contains a single fraction, the fraction can be eliminated by *multiplying both sides by the denominator of the fraction.*

**EXAMPLE:** Solve: $\frac{x}{3} - 2 = 5$

***Solution:*** Multiplying both sides by 3,

$$3\left(\frac{x}{3} - 2\right) = 3(5)$$

$$3\left(\frac{x}{3}\right) - 3(2) = 3(5)$$

$$x - 6 = 15$$

Adding 6 to both sides, $x = 15 + 6 = 21$

Check:

$$\frac{21}{3} - 2 \overset{?}{=} 5$$

$$7 - 2 = 5 \quad \text{Checks!}$$

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When there are two or more fractions, multiply both sides by the lowest common denominator to clear the fractions.

**EXAMPLE:** Solve: $\frac{x}{2} + 3 = \frac{x}{3}$

***Solution:*** Multiplying by the lowest common denominator (6),

$$6\left(\frac{x}{2} + 3\right) = 6\left(\frac{x}{3}\right)$$

$$6\left(\frac{x}{2}\right) + 6(3) = 6\left(\frac{x}{3}\right)$$

$$3x + 18 = 2x$$

Transposing, $3x - 2x = -18$

$$x = -18$$

Check:

$$\frac{-18}{2} + 3 \overset{?}{=} \frac{-18}{3}$$

$$-9 + 3 \overset{?}{=} -6$$

$$-6 = -6 \quad \text{Checks!}$$

**Strategy:** While solving an equation, you should keep in mind the objective of *getting x by itself on one side of the equal sign, with no x on the other side.*

**Steps:**

1. Eliminate fractions by multiplying both sides by the lowest common denominator.
2. Remove any parentheses by performing the indicated multiplication.
3. Like terms on the same side of the equation should be combined at any stage of the solution.
4. All terms containing *x* should be moved to one side of the equation, and all other terms moved to the other side.
5. Any coefficient of *x* may be removed by dividing both sides by that coefficient.

**EXAMPLE:** Solve: $\frac{3x-5}{2} = \frac{2(x-1)}{3} + 4$

***Solution:*** We eliminate the fractions by multiplying by 6,

$$6\left(\frac{3x-5}{2}\right) = 6\left(\frac{2(x-1)}{3}\right) + 6(4)$$

$$3(3x-5) = 4(x-1) + 6(4)$$

Removing parentheses,

$$9x - 15 = 4x - 4 + 24$$

$$9x - 15 = 4x + 20$$

We get all *x* terms on one side by adding 15 and subtracting 4*x* from both sides,

$$9x - 4x = 20 + 15$$

Combining like terms,

$$5x = 35$$

Dividing by the coefficient of *x*,

$$x = 7$$