Polynomials: Addition and Subtraction—Explanation

Adding Polynomials

To add polynomials, we add **like terms**. For a review of terms and how we add them, see the Terms Handout, #390.

Addition of polynomials problems usually appear horizontally arranged, like $(3x^2 + 2x + 3) + (4x^2 - 5x + 2)$. The parentheses can be removed immediately to get $3x^2 + 2x + 3 + 4x^2 - 5x + 2$. The task, then, is to find and add like terms; which gives us $7x^2 - 3x - 1$, which is our answer!

Example

Add the polynomials: $(x - 7) + (2x + 5) =$

$x - 7 + 2x + 5 =$ remove parentheses

$3x - 2$ add like terms

Example

Add the polynomials: $(2x + 4) + (-5x - 7) =$

$2x + 4 - 5x - 7 =$ remove parentheses

$-3x - 3$ add like terms

Example

Add the polynomials: $(3x^2 - 5) + (7x^2 - 3x + 4) =$

$3x^2 - 5 + 7x^2 - 3x + 4 =$ remove parentheses

$10x^2 - 3x - 1$ add like terms

Vertical arrangement:

These same problems can be re-written vertically with like terms aligned. This may help you to be more organized.

Example

Add the polynomials: $(x - 7) + (2x + 5) =$ re-write vertically and align like terms

$$\begin{array}{r} (x - 7) \\ \underline{+ (2x + 5)} \end{array} \Rightarrow \begin{array}{r} x - 7 \\ \underline{2x + 5} \\ 3x - 2 \end{array}$$

remove parentheses

add like terms

Example

Add the polynomials: $(3x^2 - 5) + (7x^2 - 3x + 4) =$ re-write vertically and align like terms

$$\begin{array}{r} 3x^2 \qquad - 5 \\ \underline{7x^2 - 3x + 4} \\ 10x^2 - 3x - 1 \end{array}$$

Note the missing "x" term in the first polynomial. Leave a space for it or write it in as "$0x$."

Subtracting Polynomials

You will recall the basic subtraction rule for integers which tells us to <u>add</u> the <u>opposite</u> (of what is being subtracted). Two sign changes are needed:

1. The subtraction sign changes to addition and
2. The sign (positive or negative) of each quantity being subtracted is changed to its opposite. Recall that 3 – 7 is 3 **+** (**-**7) and 2 –(-8) is 2 **+** (**+**8). This applies to subtraction of polynomials as well.

Example

Subtract the polynomials: $(3x^2 - 2x + 4) - (5x^2 + 6x - 1)$

$(3x^2 - 2x + 4) + (-5x^2 - 6x + 1)$

add opposite

$3x^2 - 2x + 4 - 5x^2 - 6x + 1$

$-2x^2 - 8x + 5$

Rewrite the problem as <u>addition</u> of the <u>opposite</u>.

We now have an <u>addition</u> of polynomials problem. Apply all you know about <u>adding</u> polynomials.

Example

Subtract the polynomials: $(7x - 8) - (4x - 6)$

$(7x - 8) + (-4x + 6)$

add opposite

$7x - 8 - 4x + 6$

$3x - 2$

Vertical arrangement:

Subtraction of polynomial problems can also be re-written vertically. Remember to account for any "missing terms" by leaving a space or indicating that there are "0" of them.

Example

Subtract the polynomials: $(3x^2 - 2x + 4) - (5x^2 + 6x - 1)$

$$\begin{array}{r} (3x^2 - 2x + 4) \\ \underline{-(5x^2 + 6x - 1)} \end{array} \Rightarrow \begin{array}{r} (3x^2 - 2x + 4) \\ \underline{+(-5x^2 - 6x + 1)} \\ -2x^2 - 8x + 5 \end{array}$$

add the opposite

Example

Subtract the polynomials: $(4x^3 - 2x + 7) - (5x^2 + 3)$

$$\begin{array}{l} (4x^3 \qquad - 2x + 7) \\ \underline{-(\quad 5x^2 \qquad + 3)} \end{array}$$

account for all missing terms
align like terms

$$\begin{array}{l} (4x^3 \qquad - 2x + 7) \\ \underline{+(\quad -5x^2 \qquad - 3)} \\ 4x^3 - 5x^2 - 2x + 4 \end{array}$$

re-write as addition of opposite
add like terms