

## STA2023 Elementary Statistics Test 2 Study Guide

Content: Chapters 4 & 5 (sections 1-4 only)

### Concepts & Skills:

1. Recognize terminology such as: *Experiment, Outcome, Sample Space, Event, Probability, Mutually Exclusive Events, Independent Events, Complementary Events, Intersection, Union, Factorial, Combination, Permutation, Random Experiment, Random Variable, Probability Distribution, Binomial Experiment, Binomial Distribution*
2. Be able to identify outcomes of an experiment and an event, identifying simple versus compound events.
3. Be able to calculate probabilities using classical probability and relative frequency probability.
4. Be able to calculate marginal, conditional, joint and union of events probabilities.
5. Be able to determine whether events are mutually exclusive, independent and complementary events.
6. Be able to determine the total number of outcomes for an experiment using the counting rule, combinations, permutations.
7. Be able to determine whether random variables are discrete or continuous.
8. Be able to identify valid probability distributions.
9. Be able to determine probabilities when given a probability distribution table.
10. Be able to construct a probability distribution table.
11. Be able to compute the mean and standard deviation of a discrete probability distribution.
12. Be able to use the binomial formula to find a probability.
13. Be able to use the binomial probabilities table to find a probability or probability distribution.
14. Be able to find the mean and standard deviation of a binomial distribution.

### Examples:

1. (C2) In a group of people, some are in favor of a tax increase on rich people to reduce the federal deficit and others are against it. (Assume that there is no other outcome such as “no opinion” and “do not know.”) Three persons are selected at random from this group and their opinions in favor or against raising such taxes are noted.
  - a. How many total outcomes are possible? Write these outcomes in a sample space  $S$ . Draw a tree diagram.
  - b. List all the outcomes included in each of the following events and mention which are simple and which are compound events.
    - i. At most one person is against a tax increase on rich people.
    - ii. Exactly two persons are in favor of a tax increase on rich people.
    - iii. At least one person is against a tax increase on rich people.
    - iv. More than one person is against a tax increase on rich people.
2. (C3) In a large city, 15,000 workers lost their jobs last year. Of them, 7400 lost their jobs because their companies closed down or moved, 4600 lost their jobs due to insufficient work, and the remainder lost their jobs because their positions were abolished. If one of these 15,000 workers is selected at random, find the probability that this worker lost his or her job
  - a. Because the company closed down or moved
  - b. Due to insufficient work
  - c. Because the position was abolished
3. (C4,C5) Two thousand randomly selected adults were asked if they think they are financially better off than their parents. The following table gives the two-way classification of the responses based on the education levels of the persons included in the survey and whether they are financially better off, the same as, or worse off than their parents.

|            | Less Than High School | High School | More Than High School |
|------------|-----------------------|-------------|-----------------------|
| Better off | 140                   | 450         | 420                   |
| Same as    | 60                    | 250         | 110                   |
| Worse off  | 200                   | 300         | 70                    |

- a. Suppose one adult is selected at random from these 2000 adults. Find the probability that
  - i. The adult is better off than their parents.
  - ii. The adult has a high school education level.
  - iii. The adult is better off than their parents given they have less than a high school education.
  - iv. The adult has more than a high school education given they are worse off than their parents.
  - v. The adult has a high school education and is the same as their parents.
  - vi. The adult is better off than their parents or has more than a high school education.
- b. Are the events less than high school education and more than high school education mutually exclusive? Are they independent?
- c. Are the events worse off and high school mutually exclusive? Are the independent?
- d. What is the complementary event of same as their parents?
- e. What is the complementary event of high school education or greater?

4. (C4) A company has installed a generator to back up the power in case there is a power failure. The probability that there will be a power failure during a snowstorm is .30. The probability that the generator will stop working during a snowstorm is .09. What is the probability that during a snowstorm the company will lose both sources of power? Note that the two sources of power are independent.
5. (C6) A ski patrol unit has nine members available for duty, and two of them are to be sent to rescue an injured skier. In how many ways can two of these nine members be selected? Now suppose the order of selection is important. How many arrangements are possible in this case?
6. (C6) A child has 6 shirts, 5 pairs of pants, 4 pairs of socks and 2 pairs of shoes. How many different outfits are possible?
7. (C6, C4) Many states have a lottery game, usually called a Pick-3, in which you pick a three-digit number. During the lottery drawing, there are three bins, each containing balls numbered 0 through 9. One ball is drawn from each bin to form the three-digit winning number.
  - a. You purchase one ticket with one three-digit number. What is the probability that you will win this lottery game?
  - b. Consider the variation that allows you to win as long as you chose the same three digits as the lottery, regardless of the order they were in. The probability of winning will depend on how many unique digits you have. Find the probability that you will win this variation of the lottery for the following:
    - i. All three digits are unique
    - ii. One of the digits appears twice
    - iii. One digit appears three times
8. (C7) Classify the following random variables according to whether they are discrete or continuous.
  - a. The number of words spelled correctly by a student on a spelling test
  - b. The number of sofas a furniture store sells monthly
  - c. The length of time an employee is late for work
  - d. The price per gallon of unleaded gas at a local service station
  - e. The yearly inflation rate in the US
  - f. The number of commercial aircraft accidents that occur per month in the US
9. (C9, C11) Medical research has shown that a certain type of chemotherapy is successful 70% of the time when used to treat skin cancer. Suppose 5 skin cancer patients are treated with this type of chemotherapy and let  $x$  equal the number of successful cures out of the 5. The probability distribution for the number  $x$  of successful cures out of 5 is given in the table:

| $x$ | $P(x)$ |
|-----|--------|
| 0   | .002   |
| 1   | .029   |
| 2   | .132   |
| 3   | .309   |
| 4   | .360   |
| 5   | .168   |

- a. Find the probability that 3 or more of the cancer patients were successfully cured.
  - b. Find the probability that no cancer patients were successfully cured.
  - c. Calculate the mean of the probability distribution and interpret its meaning.
  - d. Calculate the standard deviation of the probability distribution.
10. (C10, C11) Recent studies have found that 49% of all American households possess a gun of some kind. Three households are chosen at random, and  $x$ , the number of households possessing a gun, is counted.
    - a. Write the probability distribution.
    - b. Calculate the expected value and standard deviation of the probability distribution
  11. (C10, C11) In a group of 12 persons, 3 are left-handed. Suppose that 2 persons are randomly selected from this group. Let  $x$  denote the number of left-handed persons in this sample.
    - a. Write the probability distribution of  $x$ .
    - b. Calculate the mean and standard deviation of the probability distribution.
  12. (C12) The most recent data from the Department of Education show that 34.8% of students who submitted otherwise valid applications for a Title IV Pell Grant in 2005-2006 were ineligible to receive such a grant. Suppose that this result is true for the current population of students who submitted otherwise valid applications for this grant.
    - a. Let  $x$  be a binomial random variable that denotes the number of students in a random sample of 20 who submitted otherwise valid applications for a Title IV Pell Grant but were ineligible to receive one. What are the possible values that  $x$  can assume?
    - b. Find the probability that exactly 6 students are ineligible to receive a Title IV Pell Grant in a random sample of 20 who submitted otherwise valid applications for this grant. Use the binomial probability distribution formula.
  13. (C13, C14) Suppose that 5% of all current smartphones have Microsoft's Windows Phone operating system. Use the binomial probabilities table to write the probability distribution for a random sample of 7 smartphones.
    - a. Determine the probability that the number of smartphones that have Microsoft's Windows Phone operating system is at most 2.
    - b. Calculate the mean and standard deviation of the binomial distribution.